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LETTER TO THE EDITOR

Scaling of surface fluctuations and dynamics of surface growth models with power-law noise

Jacques G Amar and Fereydoon Family

Department of Physics, Emory University, Atlanta, GA 30322, USA

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Abstract. We report the results of extensive simulations of three surface growth models in the KPZ universality class with a power-law noise distribution of the form $P(\eta) \sim \eta^{-(\mu+1)}$ in $d = 2$. For all three models, the width and height-fluctuation distributions obey a scaling relation and exhibit power-law tails. This has implications for directly observing the presence of power-law noise in experiments. In addition, we present extensive results for both scaling exponents α and β as a function of μ for $\mu = 2$–8. Although there is some variation in the results for different models, for all three models we find anomalous exponents for $2 \leq \mu \leq 7$, while the scaling relation $\alpha + z = 2$ holds for all μ. For a linear model without sideways growth, we verify that the presence of power-law noise does not lead to anomalous exponents for $\mu > 2$.

Recently considerable progress has been made in understanding the dynamics of non-equilibrium surface growth phenomena [1] in the context of a variety of models, analytical theories, and experiments. Much of these advances stem from the fact that surface fluctuations exhibit scaling behaviour in both time and space. In particular, assuming an initially flat interface, the scaling of the interface width is expected to be of the form [2], $w(L, t) = L^{\alpha} f(t/L^{z})$, where $w(L, t)$ is the interface width on length scale L at time t, $z = \alpha/\beta$ is the dynamic exponent, and the scaling function $f(x) \sim x^{\beta}$ for $x \ll 1$ and $f(x) \to$ constant for $x \gg 1$. Analytic results based on the KPZ equation [3], and numerical simulations of related models [2, 4–7], agree in $d = 2$, indicating that $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$.

The strong universality in the evolution of rough surfaces, suggested by the agreement between analytical and numerical results, has been questioned by the results of recent experiments on two-fluid flow in porous media [8–11] and on the growth of bacterial colonies [12], which yield significantly higher values for α and β in $d = 2$. In particular, a recent experiment in porous media gives [11] $\alpha \simeq 0.81$, and $\beta \simeq 0.625$. Similarly, in the experiment on bacterial growth, Vicsek *et al* [12] find $\alpha = 0.78 \pm 0.07$. Although these exponents are significantly larger than the values obtained previously for simple growth models, they approximately satisfy the scaling identity [5, 13, 14] $\alpha + z = 2$, which applies to models in the KPZ universality class [3].

A possible explanation for the anomalously large exponents found in these experiments has been recently proposed by Zhang [15]. He suggested that the new exponents arise from the fact that the amplitude of the random noise in the experiments has a non-Gaussian, power-law distribution of the form $P(\eta(\boldsymbol{r}, t)) \sim 1/\eta^{1+\mu}$, where $\eta(\boldsymbol{r}, t)$ is the delta-correlated noise. From simulations of a discrete model with non-Gaussian noise in $d = 2$, he obtained preliminary results for the exponent α for $\mu = 2$–7, which

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was found to decrease from a value of 1 at $\mu=2$ towards the Gaussian noise value of $\frac{1}{2}$ with increasing μ. The result $\alpha \simeq 0.75$, for $\mu=3$, is relatively close to the experimental results. The possibility that this model could explain the experimental results as well as the possibility of a new universality class for the KPZ equation is quite intriguing. Therefore more extensive investigations of this model and its variants may provide a deeper insight into the experiments as well as various theoretical approaches to surface growth.

In this letter we present the results of extensive studies in $d=2$ of the scaling properties of several surface growth models which belong to the universality class of the KPZ equation with non-Gaussian noise. Since experiments directly measure not only the average width and height of the interface but also the surface profile, we have also investigated the distribution of the width and height fluctuations of the interface at saturation. In addition, we have conducted an extensive study of both scaling exponents for each model for $\mu=2$ to 8. For all models studied, the KPZ scaling relation $\alpha+z=2$ is found to hold, while no crossover to Gaussian exponents is observed.

The first model we studied is the Zhang model [15] of interface growth for which the evolution of the surface is defined by the discrete equation,

$$h(i, t+1)=\max(h(i-1, t)+\eta(i-1, t), h(i+1, t)+\eta(i+1, t)) \tag{1}$$

where $h(i, t)$ is the height of the interface at position i at time t, and i runs over only even indices if t is even, odd indices if t is odd, and the noise η has the distribution,

$$\begin{aligned} &P(\eta) \sim \frac{1}{\eta^{1+\mu}} \qquad \text{for } \eta>1 \\ &P(\eta)=0 \qquad \text{otherwise.} \end{aligned} \tag{2}$$

This model is believed to be equivalent [15, 16] to the $T=0$ directed polymer in a random potential and accordingly to a discrete-time KPZ equation. For $0 \leqslant \mu \leqslant 2$, the first and second moments of $P(\eta)$ diverge, and therefore anomalous exponents are not surprising. For $\mu>2$, due to the central-limit theorem, the average of the noise in the continuum limit has a Gaussian distribution and thus one might expect a crossover to Gaussian exponents. However, it has been suggested by Zhang [17] that since this model is equivalent to a discrete-time KPZ equation, no continuum limit exists and thus anomalous exponents are possible even for $\mu>2$.

In addition to results for this model, we also present results for two other growth models with non-Gaussian noise. The second model studied is a noise-enhanced variation of the Zhang model in which successive sites on the same sublattice do not see the same noise at the site in between them. The third model is a modification of the ballistic deposition model [18], for which the evolution equation is,

$$h(i, t+1)=\max(h(i-1, t), h(i+1, t), h(i, t)+\eta(i, t)) \tag{3}$$

where again η obeys equation (2).

For each of the growth models studied, the noise η was generated at each odd (even) site at each odd (even) time step t, by generating an independent random number r at each site such that $0<r<1$ and calculating the quantity $\eta=r^{-(1/\mu)}$. We have recently shown [19] that a finite cut-off in the distribution leads to a crossover to Gaussian noise behaviour. Therefore, we used both 32-bit and 64-bit random numbers in order to ensure that an artificial cut-off was not introduced into the distribution.

Since experiments directly measure not only the average width and height of the interface but also the surface profile, we have first determined the distribution of the surface width w and the fluctuations in the height $\delta h(\boldsymbol{r}, t) = h(\boldsymbol{r}, t) - \langle h(\boldsymbol{r}, t)\rangle_r$, as a function of system size L at saturation for different μ. We find that both distributions can be scaled for all values of μ using the scaling ansatz,

$$P(L, X) \sim L^{-\alpha} F(L^{-\alpha} X) \tag{4}$$

where $X = w$ or δh. This scaling form was previously found for surface growth models with Gaussian noise [20]. Figures 1 and 2 show typical scaling plots of this form for the Zhang model with $\mu = 3$ for the width distribution and the height-fluctuation distribution respectively. Similar plots were obtained for other values of μ as well as for the other two models. In all cases we find that both distributions and the corresponding scaling functions have power-law tails of the form $u^{-(\mu+1)}$. This implies that the existence of power-law noise may be determined experimentally from measurements of these distributions. We note that for the case of power-law noise the scaling form (4) leads to the scaling relation, $\langle (X_L)^n \rangle \sim L^{n\alpha}$ for $n < \mu$, which implies that there is no multiscaling behaviour for the quantities w and δh for $n < \mu$.

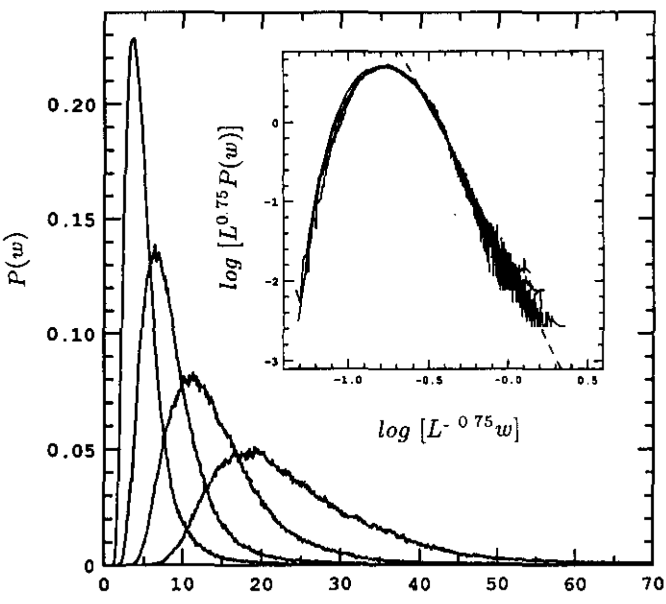

Figure 1. Width distribution function $P(L, w)$ at $\mu = 3$ for four different values of L (L ranges from 64 to 512 in powers of 2). Inset shows scaling plot with $\alpha = 0.75$ for data in main figure. Slope of broken line is -4.0.

We now discuss our results for the scaling exponents α and β. Figure 3 shows data for the Zhang model for the saturation width (RMS deviation of the surface height) against system size L for different values of μ, as well as the fits used to determine the exponent α. Averages for all system sizes were taken over times of the order of several million time steps, i.e. significantly longer than the correlation time for even the largest L. Similar results for the noise-enhanced version of the Zhang model yield

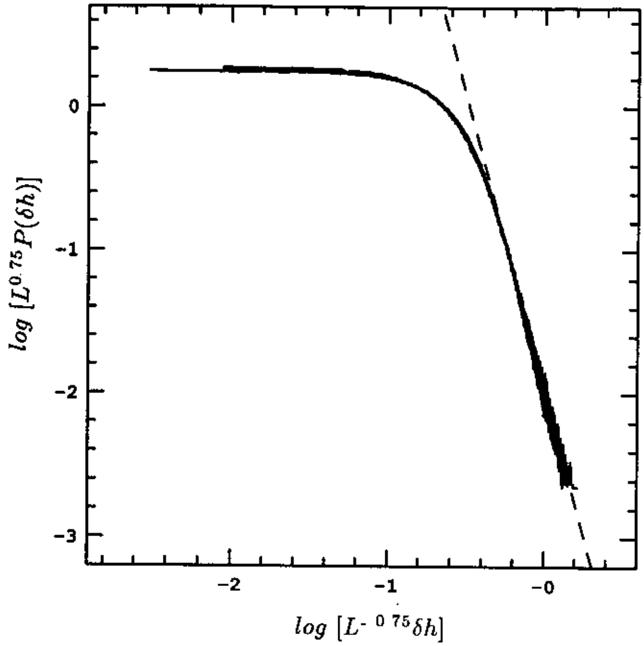

Figure 2. Scaling plot of form of (4) for the height-fluctuation distribution $P(\delta h)$ for $L = 64$, 128, and 256. Since $P(\delta h)$ is symmetric about 0, only distribution for $\delta h > 0$ is shown. Broken line has slope -4.0.

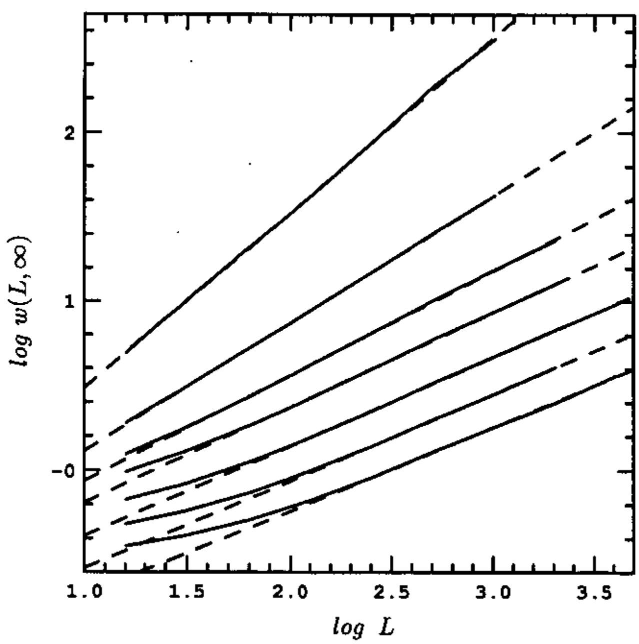

Figure 3. Log-log plots of saturation width $w(L,\infty)$ against L for the Zhang model, for $\mu = 2$-8, and $L = 16$ to 4096. Top curve is for $\mu = 2$, bottom is for $\mu = 8$. Broken lines show fits to data with slopes ($\mu = 2$ to 8) 1.04, 0.756, 0.625, 0.56, 0.525, 0.51, 0.50. Bottom three curves have been shifted down by 0.1, 0.2, and 0.3 respectively for clarity.

slightly higher values of α. We note that for both models the crossover to asymptotic scaling occurs at larger L with increasing μ. In addition, no sign of a crossover to Gaussian exponents is observed.

Figure 4 shows similar data for the ballistic deposition model defined by (3). For this model, we find results for the exponent α which are somewhat larger than obtained for the Zhang model. In addition, crossover effects appear to be significantly smaller in this model, although the slope appears to be decreasing slightly with system size L.

Figure 5 shows data for the early-time behaviour for each value of μ for the Zhang model. Values for the growth exponent β were determined directly from runs of the order of 2000 to 20 000 time steps on very large systems from $L = 131\,072$ to $L = 524\,288$, with averages over 20 to 100 runs. As in previous studies [20, 21] of surface growth, self-averaging of the surface width was observed to hold beyond the earliest times for $\mu > 3$. For $\mu = 2$ and 3, however, large fluctuations were observed and the surface width appeared to be only weakly self-averaging†. In addition, the growth exponent β appeared to be very sensitive to the effects of finite system size. Thus, for $\mu > 3$ the simulation of very large systems at early times was statistically equivalent to many

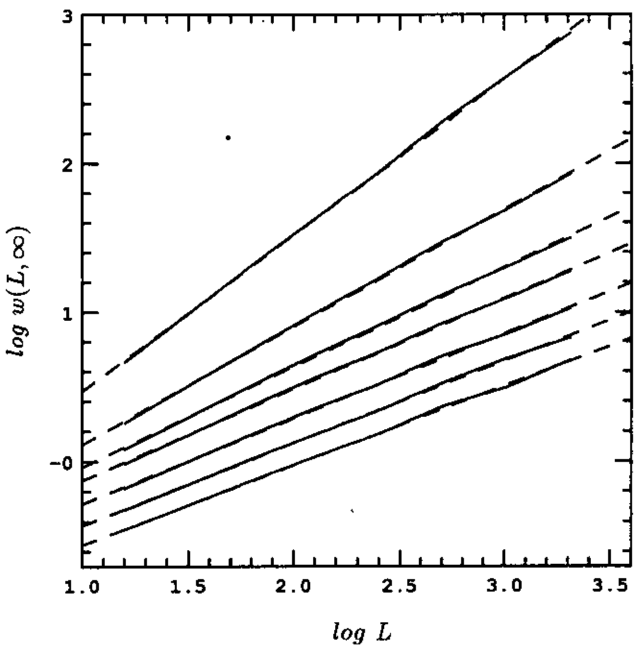

Figure 4. Log-log plots of saturation width $w(L, \infty)$ against L for the ballistic deposition model for $\mu = 2-8$, $L = 16$ to 2048. Top curve is for $\mu = 2$, bottom is for $\mu = 8$. Broken lines show fits to data with slopes ($\mu = 2$ to 8), 1.03, 0.77, 0.65, 0.59, 0.555, 0.54, 0.53. Bottom three curves have been shifted down by 0.1, 0.2, and 0.3 respectively for clarity.

† We note that the quantity $w = \langle|\delta h|\rangle$ (which is an equally valid measure of the width) was found to be self-averaging for all μ studied. Similar differences in the self-averaging properties were also found for different definitions of the height-height correlation function for $\mu = 2$ and 3. Details of this work will be reported elsewhere.

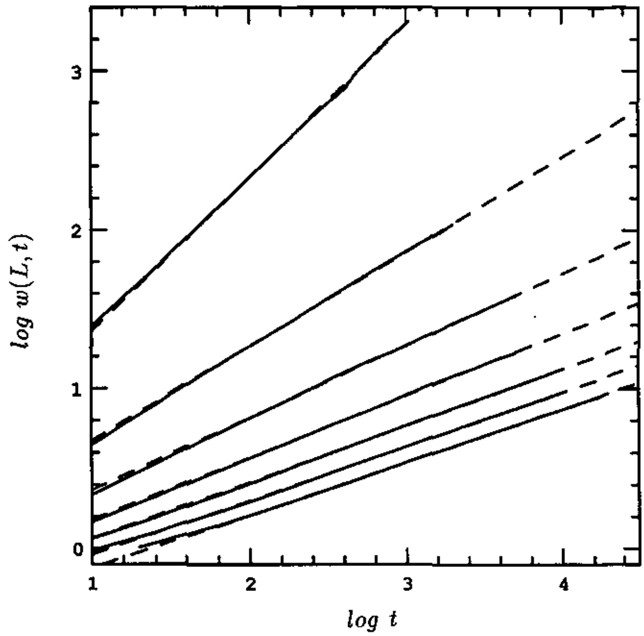

Figure 5. Log-log plots of $w(L, t)$ against t for the Zhang model for $\mu = 2$–8. Broken lines show fits to data with slopes ($\mu = 2$ to 8) 0.98, 0.60, 0.449, 0.395, 0.355, 0.339, 0.331.

more runs on smaller systems, and allowed us to avoid the saturation effects due to system size. Our results (see figure 5) for the exponent β for the Zhang model for $\mu > 2$ satisfy very accurately (within 0.02) the KPZ scaling relation $\alpha + \alpha/\beta = 2$. For $\mu = 2$ the scaling relation was not found to hold as accurately ($\alpha + \alpha/\beta = 2.1$). Similar data were obtained for the ballistic deposition model. The values obtained for β are again somewhat larger than for the Zhang model. We note, however, that the KPZ scaling relation was found to hold less accurately for this model.

Figure 6 summarizes our results for the exponent α for all three models, for $2 \leqslant \mu \leqslant 8$. As already mentioned, for the ballistic deposition model defined by (3) the exponents are somewhat larger for all μ than for the other two models. At the moment we do not completely understand the reason behind this discrepancy. However, since the relation $\alpha + z = 2$ is approximately satisfied by all three models, it is reasonable to assume that they all belong to the KPZ universality class. Therefore, in the asymptotic limit they should agree. In this connection we note that results for the Zhang model for the exponent α, obtained from the scaling of the saturation velocity as a function of system size [19], yielded somewhat higher values than obtained from the scaling of the width w for $\mu \geqslant 4$. (Specifically, we obtained for $\mu = 3-8$, $\alpha = 0.74$, 0.63, 0.58, 0.55, 0.535, 0.53.) In addition, better scaling behaviour at small L (than for the width w) was observed. Thus, more work is needed to determine if the discrepancy is simply a crossover effect or is in fact an indication of a breakdown of subuniversality.

One interesting question regarding the universality class of the KPZ equation with power-law noise, is whether there exists a critical value of $\mu = \mu_c$ beyond which the exponents cross over to Gaussian noise behaviour in $d = 2$. Our results for all models indicate that for $\mu \leqslant 7$, the exponents α and β remain clearly above the Gaussian values $\alpha = \frac{1}{2}$, $\beta = \frac{1}{3}$. However at $\mu = 8$, for both versions of the Zhang model, we obtain

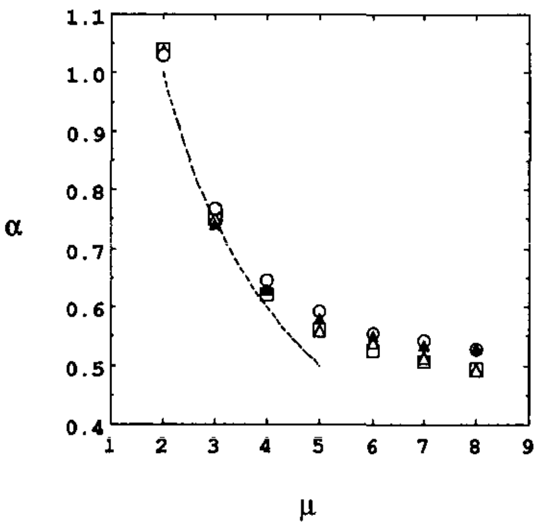

Figure 6. Surface roughness exponent α as a function of μ for all three models. Open squares and triangles are results for the Zhang model and its noise-enhanced variant respectively, while circles are results for model of (3). Full triangles are estimates for the Zhang model based on fits to the scaling of the saturation velocity with system size. Broken curve is Flory-theory prediction in $d=2$.

results which are essentially indistinguishable within error bars from the Gaussian values. We note that our results for α and β for all three models disagree strongly with the Flory-type formula [17, 22] $\alpha=(1+d)/(\mu+1)$ which predicts $\mu_c=5$ in $d=2$.

As an additional check that the anomalous results obtained for $\mu>2$ for the exponents α and β are not due to crossover effects, we also studied a linear growth model for which the addition of non-Gaussian noise is not expected to affect the asymptotic values of the exponents. For this model, the growth rule is that the new height $h(i, t+1)$ is equal to the average of itself and its two nearest neighbours at time t, plus an additional noise term. A study of the growth exponents for this model, which is in the Edwards-Wilkinson [23] universality class, confirms that the presence of power-law noise does not alter the growth exponents i.e. $\alpha=\frac{1}{2}$, $\beta=\frac{1}{4}$, and $z=2$ for $\mu>2$. *In addition, no unusual crossover effects were seen for* $\mu>2$.

In conclusion, we have studied three different surface growth models with power-law noise in $d=2$. For all three models, we find that the distributions of the width and the height fluctuations obey a scaling form with a power-law tail, thus providing a novel technique for determining the existence of non-Gaussian noise in experiments on surface growth. In addition, we find that the scaling exponents α and β are anomalous for $2\leqslant\mu\leqslant 7$ for all three models. While some discrepancies are observed for the values of the exponents, we believe this is most likely due to crossover effects. Finally, we have verified that for a model without sideways growth, the presence of power-law noise does not alter the exponents or cause anomalous crossover behaviour for $\mu>2$. This provides further support for the existence of anomalous exponents for models in the KPZ universality class with power-law noise.

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